Package 'basefun'

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Title Infrastructure for Computing with Basis Functions

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Description Some very simple infrastructure for basis functions.

Depends variables (>= 1.1-0), R (>= 3.2.0)

Imports stats, polynom, Matrix, orthopolynom, methods

Suggests coneproj

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R topics documented:

basefun-package *General Information on the* **basefun** *Package*

Description

The **basefun** package offers a small collection of objects for handling basis functions and corresponding methods.

The package was written to support the **mlt** package and will be of limited use outside this package.

Author(s)

This package is authored by Torsten Hothorn <Torsten.Hothorn@R-project.org>.

References

Torsten Hothorn (2018), Most Likely Transformations: The mlt Package, *Journal of Statistical Software*, forthcoming. URL: https://cran.r-project.org/package=mlt.docreg

as.basis *Convert Formula or Factor to Basis Function*

Description

Convert a formula or factor to basis functions

Usage

```
as.basis(object, ...)
## S3 method for class 'formula'
as.basis(object, data = NULL, remove_intercept = FALSE,
         ui = NULL, ci = NULL, negative = FALSE, scale = FALSE,
         Matrix = FALSE, prefix = "", ...)
## S3 method for class 'factor_var'
as.basis(object, ...)
## S3 method for class 'ordered_var'
as.basis(object, ...)
## S3 method for class 'factor'
as.basis(object, ...)
## S3 method for class 'ordered'
as.basis(object, ...)
```

Arguments

object	a formula or an object of class factor, factor_var, ordered or ordered_var
data	either a vars object or a data.frame
remove_intercept	a logical indicating if any intercept term shall be removed
ui	a matrix defining constraints
ci	a vector defining constraints
negative	a logical indicating negative basis functions
scale	a logical indicating a scaling of each column of the model matrix to the unit interval (based on observations in data)
Matrix	a logical requesting a sparse model matrix, that is, a Matrix object.
prefix	character prefix for model matrix column names (allows disambiguation of parameter names).
...	additional arguments to model.matrix, for example contrasts

Details

as.basis returns a function for the evaluation of the basis functions with corresponding model.matrix and predict methods.

Unordered factors (classes factor and factor_var) use a dummy coding and ordered factor (classes ordered or ordered_var) lead to a treatment contrast to the last level and removal of the intercept term with monotonicity constraint. Additional arguments (...) are ignored for ordered factors.

Linear constraints on parameters parm are defined by ui %*% parm >= ci.

Examples

```
## define variables and basis functions
v <- c(numeric_var("x"), factor_var("y", levels = LETTERS[1:3]))
fb <- as.basis(~ x + y, data = v, remove_intercept = TRUE, negative = TRUE,
               contrasts.arg = list(y = "contr.sum"))

## evaluate basis functions
model.matrix(fb, data = as.data.frame(v, n = 10))
## basically the same as (but wo intercept and times -1)
model.matrix(~ x + y, data = as.data.frame(v, n = 10))

### factor
xf <- gl(3, 1)
model.matrix(as.basis(xf), data = data.frame(xf = xf))

### ordered
xf <- gl(3, 1, ordered = TRUE)
model.matrix(as.basis(xf), data = data.frame(xf = unique(xf)))
```

b *Box Product of Basis Functions*

Description

Box product of two basis functions

Usage

```
b(..., sumconstr = FALSE)
```

Arguments

...	named objects of class basis
sumconstr	a logical indicating if sum constraints shall be applied

Details

b() joins the corresponding design matrices by the row-wise Kronecker (or box) product.

Examples

```
### set-up a Bernstein polynomial
xv <- numeric_var("x", support = c(1, pi))
bb <- Bernstein_basis(xv, order = 3, ui = "increasing")
## and treatment contrasts for a factor at three levels
fb <- as.basis(~ g, data = factor_var("g", levels = LETTERS[1:3]))

### join them: we get one intercept and two deviation _functions_
bfb <- b(bern = bb, f = fb)

### generate data + coefficients
x <- expand.grid(mkgrid(bfb, n = 10))
cf <- c(1, 2, 2.5, 2.6)
cf <- c(cf, cf + 1, cf + 2)

### evaluate bases
model.matrix(bfb, data = x)

### plot functions
plot(x$x, predict(bfb, newdata = x, coef = cf), type = "p",
     pch = (1:3)[x$g])
legend("bottomright", pch = 1:3,
       legend = colnames(model.matrix(fb, data = x)))
```

Bernstein_basis *Bernstein Basis Functions*

Description

Basis functions defining a polynomial in Bernstein form

Usage

```
Bernstein_basis(var, order = 2, ui = c("none", "increasing", "decreasing",
                                       "cyclic", "zerointegral", "positive",
                                       "negative", "concave", "convex"),
                extrapolate = FALSE, log_first = FALSE)
```

Arguments

var	a numeric_var object
order	the order of the polynomial, one defines a linear function
ui	a character describing possible constraints
extrapolate	logical; if TRUE, the polynomial is extrapolated linearily outside support(var). In particular, the second derivative of the polynomial at support(var) is constrained to zero.
log_first	logical; the polynomial in Bernstein form is defined on the log-scale if TRUE. It makes sense to define the support as c(1, q)$, ie putting the first basis function of the polynomial on log(1).

Details

Bernstein_basis returns a function for the evaluation of the basis functions with corresponding model.matrix and predict methods.

References

Rida T. Farouki (2012), The Bernstein Polynomial Basis: A Centennial Retrospective, *Computer Aided Geometric Design*, **29**(6), 379–419. http://dx.doi.org/10.1016/j.cagd.2012.03.001

Examples

```
### set-up basis
bb <- Bernstein_basis(numeric_var("x", support = c(0, pi)),
                      order = 3, ui = "increasing")

### generate data + coefficients
x <- as.data.frame(mkgrid(bb, n = 100))
cf <- c(1, 2, 2.5, 2.6)
```

```
### evaluate basis (in two equivalent ways)
bb(x[1:10,,drop = FALSE])
model.matrix(bb, data = x[1:10, ,drop = FALSE])

### check constraints
cnstr <- attr(bb(x[1:10,,drop = FALSE]), "constraint")
all(cnstr$ui %*% cf > cnstr$ci)

### evaluate and plot Bernstein polynomial defined by
### basis and coefficients
plot(x$x, predict(bb, newdata = x, coef = cf), type = "l")

### evaluate and plot first derivative of
### Bernstein polynomial defined by basis and coefficients
plot(x$x, predict(bb, newdata = x, coef = cf, deriv = c(x = 1)),
     type = "l")

### illustrate constrainted estimation by toy example
N <- 100
order <- 10
x <- seq(from = 0, to = pi, length.out = N)
y <- rnorm(N, mean = -sin(x) + .5, sd = .5)

if (require("coneproj")) {
  prnt_est <- function(ui) {
    xv <- numeric_var("x", support = c(0, pi))
    xb <- Bernstein_basis(xv, order = 10, ui = ui)
    X <- model.matrix(xb, data = data.frame(x = x))
    uiM <- as(attr(X, "constraint")$ui, "matrix")
    ci <- attr(X, "constraint")$ci
    if (all(is.finite(ci)))
      parm <- qprog(crossprod(X), crossprod(X, y),
                    uiM, ci, msg = FALSE)$thetahat
    else
      parm <- coef(lm(y ~ 0 + X))
    plot(x, y, main = ui)
    lines(x, X %*% parm, col = col[ui], lwd = 2)
  }
  ui <- eval(formals(Bernstein_basis)$ui)
  col <- 1:length(ui)
  names(col) <- ui
  layout(matrix(1:length(ui),
                ncol = ceiling(sqrt(length(ui)))))
  tmp <- sapply(ui, function(x) try(prnt_est(x)))
}
```

`c.basis` *Join Basis Functions*

Description

Concatenate basis functions column-wise

Usage

```
## S3 method for class 'basis'
c(..., recursive = FALSE)
```

Arguments

`...`	named objects of class `basis`
`recursive`	always `FALSE`

Details

`c()` joins the corresponding design matrices column-wise, ie, the two functions defined by the two bases are added.

Examples

```
### set-up Bernstein and log basis functions
xv <- numeric_var("x", support = c(1, pi))
bb <- Bernstein_basis(xv, order = 3, ui = "increasing")
lb <- log_basis(xv, remove_intercept = TRUE)

### join them
blb <- c(bern = bb, log = lb)

### generate data + coefficients
x <- as.data.frame(mkgrid(blb, n = 100))
cf <- c(1, 2, 2.5, 2.6, 2)

### evaluate bases
model.matrix(blb, data = x[1:10, ,drop = FALSE])

### evaluate and plot function defined by
### bases and coefficients
plot(x$x, predict(blb, newdata = x, coef = cf), type = "l")

### evaluate and plot first derivative of function
### defined by bases and coefficients
plot(x$x, predict(blb, newdata = x, coef = cf, deriv = c(x = 1)),
     type = "l")
```

`intercept_basis` *Intercept-Only Basis Function*

Description

A simple intercept as basis function

Usage

```
intercept_basis(ui = c("none", "increasing", "decreasing"), negative = FALSE)
```

Arguments

ui	a character describing possible constraints
negative	a logical indicating negative basis functions

Details

`intercept_basis` returns a function for the evaluation of the basis functions with corresponding `model.matrix` and `predict` methods.

Examples

```
### set-up basis
ib <- intercept_basis()

### generate data + coefficients
x <- as.data.frame(mkgrid(ib))

### 2 * 1
predict(ib, newdata = x, coef = 2)
```

Legendre_basis *Legendre Basis Functions*

Description

Basis functions defining a Legendre polynomial

Usage

```
Legendre_basis(var, order = 2, ui = c("none", "increasing", "decreasing",
                                      "cyclic", "positive", "negative"), ...)
```

Arguments

var	a numeric_var object
order	the order of the polynomial, one defines a linear function
ui	a character describing possible constraints
...	additional arguments passed to legendre.polynomials

Details

`Legendre_basis` returns a function for the evaluation of the basis functions with corresponding `model.matrix` and `predict` methods.

References

Rida T. Farouki (2012), The Bernstein Polynomial Basis: A Centennial Retrospective, *Computer Aided Geometric Design*, **29**(6), 379–419. http://dx.doi.org/10.1016/j.cagd.2012.03.001

Examples

```
### set-up basis
lb <- Legendre_basis(numeric_var("x", support = c(0, pi)),
                     order = 3)

### generate data + coefficients
x <- as.data.frame(mkgrid(lb, n = 100))
cf <- c(1, 2, 2.5, 1.75)

### evaluate basis (in two equivalent ways)
lb(x[1:10,,drop = FALSE])
model.matrix(lb, data = x[1:10, ,drop = FALSE])

### evaluate and plot Legendre polynomial defined by
### basis and coefficients
plot(x$x, predict(lb, newdata = x, coef = cf), type = "l")
```

---

log_basis *Logarithmic Basis Function*

Description

The logarithmic basis function

Usage

```
log_basis(var, ui = c("none", "increasing", "decreasing"),
          remove_intercept = FALSE)
```

Arguments

`var`	a `numeric_var` object
`ui`	a character describing possible constraints
`remove_intercept`	a logical indicating if the intercept term shall be removed

Details

log_basis returns a function for the evaluation of the basis functions with corresponding model.matrix and predict methods.

Examples

```
### set-up basis
lb <- log_basis(numeric_var("x", support = c(0.1, pi)))

### generate data + coefficients
x <- as.data.frame(mkgrid(lb, n = 100))

### 1 + 2 * log(x)
max(abs(predict(lb, newdata = x, coef = c(1, 2)) - (1 + 2 * log(x$x))))
```

---

polynomial_basis *Polynomial Basis Functions*

Description

Basis functions defining a polynomial

Usage

```
polynomial_basis(var, coef, ui = NULL, ci = NULL)
```

Arguments

var	a numeric_var object
coef	a logical defining the order of the polynomial
ui	a matrix defining constraints
ci	a vector defining constraints

Details

polynomial_basis returns a function for the evaluation of the basis functions with corresponding model.matrix and predict methods.

Examples

```
### set-up basis of order 3 ommiting the quadratic term
pb <- polynomial_basis(numeric_var("x", support = c(0, pi)),
                       coef = c(TRUE, TRUE, FALSE, TRUE))

### generate data + coefficients
```

```
x <- as.data.frame(mkgrid(pb, n = 100))
cf <- c(1, 2, 0, 1.75)

### evaluate basis (in two equivalent ways)
pb(x[1:10,,drop = FALSE])
model.matrix(pb, data = x[1:10, ,drop = FALSE])

### evaluate and plot polynomial defined by
### basis and coefficients
plot(x$x, predict(pb, newdata = x, coef = cf), type = "l")
```

`predict.basis` *Evaluate Basis Functions*

Description

Evaluate basis functions and compute the function defined by the corresponding basis

Usage

```
## S3 method for class 'basis'
predict(object, newdata, coef, dim = !is.data.frame(newdata), ...)
## S3 method for class 'cbind_bases'
predict(object, newdata, coef, dim = !is.data.frame(newdata),
         terms = names(object), ...)
## S3 method for class 'box_bases'
predict(object, newdata, coef, dim = !is.data.frame(newdata), ...)
```

Arguments

`object`	a `basis` or `bases` object
`newdata`	a `list` or `data.frame`
`coef`	a vector of coefficients
`dim`	either a logical indicating that the dimensions shall be obtained from the `bases` object or an integer vector with the corresponding dimensions (the latter option being very experimental
`terms`	a character vector defining the elements of a `cbind_bases` object to be evaluated
`...`	additional arguments

Details

`predict` evaluates the basis functions and multiplies them with `coef`. There is no need to expand multiple variables as `predict` uses array models (Currie et al, 2006) to compute the corresponding predictions efficiently.

References

Ian D. Currie, Maria Durban, Paul H. C. Eilers, P. H. C. (2006), Generalized Linear Array Models with Applications to Multidimensional Smoothing, *Journal of the Royal Statistical Society, Series B: Methodology*, **68**(2), 259–280.

Examples

```
### set-up a Bernstein polynomial
xv <- numeric_var("x", support = c(1, pi))
bb <- Bernstein_basis(xv, order = 3, ui = "increasing")
## and treatment contrasts for a factor at three levels
fb <- as.basis(~ g, data = factor_var("g", levels = LETTERS[1:3]))

### join them: we get one intercept and two deviation _functions_
bfb <- b(bern = bb, f = fb)

### generate data + coefficients
x <- mkgrid(bfb, n = 10)
cf <- c(1, 2, 2.5, 2.6)
cf <- c(cf, cf + 1, cf + 2)

### evaluate predictions for all combinations in x (a list!)
predict(bfb, newdata = x, coef = cf)

## same but slower
matrix(predict(bfb, newdata = expand.grid(x), coef = cf), ncol = 3)
```

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